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The Value of Weather Risk Under Ambiguity, Insurance and Self-Protection

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A B S T R A C T

The weather-sensitive industries exhibit large differences in revenue between normal and bad years. This study develops a two-state model to formulize the value of weather risk (VWR) based on maximizing individual revenue. Large VWR is worth investing more in disaster risk reduction. The analytic expression of VWR measures the optimal tradeoff between input reduction (decreasing future revenue) and risk reduction (decreasing possible loss). As risk ambiguity and decision maker's ambiguity aversion increase, VWR will increase. The use of insurance or self-protection can decrease VWR under concave revenue functions, but is inefficient under convex ones. We further generalize the model to evaluate the average VWR of all individuals, or apply it to compute other disaster risk value.

Keywords: the value of weather risk, ambiguity aversion, self-protection, weather risk management, weather forecast

JEL classification: D81, Q51, I18 and C10

1. Introduction

Weather risks have considerable impact on some industries highly sensitive to minor weather changes, such as energy, retail, agriculture, clothing, tourism, transportation, and construction industries. Thirty percent of gross domestic product of the United States is affected by the weather. The demand for weather risk management increases when these industries desire improved cash flow stability, more accurate budget management, greater earnings consistency, and higher risk-adjusted returns. When decision-makers (DMs) determine how much budget for disaster prevention (decreasing possible loss) or production input (increase revenue), benefit-cost analysis in weather risk management is necessary because these two budgets are crowded out each other. In order to maximize the firm’s wealth, a DM should spend how much in reducing per unit risk, which is defined by the value of weather risk (VWR) in this study. Intuitively, the higher the VWR, the higher DMs are willing to pay for risk reduction. Just as the value of a statistical life is a key parameter in the analysis of government policy (Bleichrodt, et al, 2019), VWR is a key parameter in weather risk management of a firm.

The seasonal weather is predictable by probabilistic forecasts but not exact. There exists the ambiguity with respect to the distribution of weather. The technology used for modern weather predictions combines computer-based models with human experience: for instance, the El Niño–Southern Oscillation, linked to variations in seasonal forecasts (Goddard et al., 2001), typically occurs every 3–5 years and typically lasts 2 years. However, historically, the occurrence interval has varied from 2 to 7 years and has lasted as long as 3–4 years. The presence of systemic weather risks (Miranda and Glauber, 1997) lead to many challenges in efficiently and accurately forecasting business revenues, including restating input source, and modeling geographical differences across regions. Carriquiry and Osgood (2012) studied the interaction among weather forecasts, index insurance and input decisions. They do not consider the ambiguity in probabilistic weather forecasts and the ambiguity aversion of decision maker. Since the Ellsberg (1961) paradox, many experimental works have demonstrated that subjects are not indifferent to ambiguity over probabilities (Camerer, 1995). Various decision models with forms of ambiguity aversion have been proposed by decision theorists (Etner et al., 2012). Therefore, this study fulfills the gap in the literature. Our model incorporates business revenues, weather forecast and decision makers’ ambiguity aversion to the formula of VWR.

Weather forecasts developed by climatologists are for reference, and DMs must consider ambiguity[1] in weather risk, including occurrence probability and loss distribution when they make policies to maximize a (weather-dependent) expected benefit. This paper refers to the theory of ambiguity aversion developed by Klibanoff et al. (2005), and the effect of ambiguity aversion on the VWR is analyzed in this study.

Action against weather risk may reduce the probability of loss occurrence (i.e., self-protection) or reduce the severity of a potential loss (i.e., insurance). Insurance costs DMs in all states to increase wealth in a loss state. This method reduces revenue volatility. In practice, most DMs are ambiguity averse. Their willingness to pay for insurance increases with ambiguity and ambiguity aversion demonstrates that ambiguity aversion increases the demand for insurance (Snow, 2011; Alary et al., 2013). This study shows that ambiguity averters use insurance to reduce the VWR, and further examines the criterion for adopting insurance.

Self-protection aims at reducing the probability of a loss state at a cost incurred in all states. This method may strengthen protection against risks or improve weather forecasts to avoid more input under a bad weather. Accurate forecasts reduce weather risks and simultaneously reduced weather ambiguity.

[1] Knight (1921) defines ambiguity as uncertainty about probability, created by missing information that is relevant and could be known.

Without accurate forecasts, the probability of earning insufficient revenue is almost equal to that of having a bad weather. If the weather forecast is effective, then the probability of earning insufficient revenue is less than that of having a bad weather. DMs can make smart inputs to get more or lose less. Snow (2010) reveals that ambiguity aversion increases the demand for self-protection, but Alary et al. (2013) arrive at the contradictory conclusion. Treich (2010) argues that the effect of ambiguity aversion on self-protection is unclear. In the current study, we demonstrate that self-protection reduces the VWR, and further examine the criterion for adopting self-protection.

The contributions of this paper are as follows: First, we formulize the VWR on the basis of different revenues in normal and bad weathers and show that the VWR increases with ambiguity aversion. Benefit–cost analysis thus can be employed to determine how to manage weather risk. The VWR also refers to the maximum payment for reducing one unit risk. This study demonstrates that the VWR is derived from the subjective value of ambiguity aversion and the objective value of the revenue difference (between in normal years and in bad years) and revenue margins. Second, compared to past literature, our model first adopts different revenue functions instead of concave utility function. The revenue functions may be concave or convex at different production stages. The use of insurance and self-protection against weather risk may be not efficient at the convex production stage. We examine the contradictory use of insurance and self-protection criteria, and demonstrate that both insurance and self-protection can reduce the VWR. The value of information for forecasting weather depends on the forecast accuracy and revenue differences. Finally, we generalize the model for more applications and examine the model's limits. The discussion and analysis of such model can contribute to an efficient and cost effective risk management approach, and then help reduce risk exposure and vulnerability.

The remainder of the paper is organized as follows: Section 2 discusses and formulizes the VWR under ambiguity aversion, insurance, and self-protection conditions. Section 3 generalizes the model and explains the model limit. The conclusions are presented in Section 4.

2. VWR under ambiguity aversion, insurance, and self-protection

To greatly simplify the technical analysis, we assume that the input wealth w results in a high revenue amount $u(w)$ in a normal weather (year) with the probability 1-p, and a low revenue amount $v(w)$ in a bad weather (year) with the probability p, where $u(w) > w > v(w) > 0$. A DM may purchase an insurance policy with special premium τ and coverage I. The revenues are adjusted by in a high revenue amount $u(w-\tau)$ in a normal weather (year) and a low revenue amount $v(w-\tau)+I$ in a bad weather (year). A DM may pay some expense e for disaster resistance to reduce this probability $p(e)$ of bad weather. The expected revenues are adjusted by $(1-p(e))\,u(w-e)$ in a normal weather (year) and $p(e)\,v\,(w-e)$ in a bad weather (year).

As economists often assume that a firm's production function is increasing and concave, the first and second derivations of two revenue functions $u(w)$ and $v(w)$ are assumed to follow $u'(w) > v'(w) > 0$ and $0 > u''(w) > v''(w)$. The input w is considered product cost. This is logical because DMs can obtain profit $u(w)-w$ (which means revenue minus cost) in a normal year or suffer loss $w-v(w)$ in a bad year. Individual (state-dependent) expected benefit B is given using

$$B=(1-p)u(w)+pv(w)-w. \tag{1}$$

The expected benefit is the expected revenue (of normal and bad years) minus the input wealth. Each DM may have idiosyncratic normal revenue $u(w)$ and bad revenue $v(w)$, and faces various weather risks. We expand the model to a general form Section 4.

According to Eq. (1), Proposition 2.1 provides an explicit formula of the VWR:

Proposition 2.1. The VWR is expressed as

$$\text{VWR} \equiv \frac{dw}{dp} = \frac{u(w)-v(w)}{(1-p)u'(w)+pv'(w)-1}. \tag{2}$$

Proof: A DM maximizes a (state-dependent) expected benefit. The first condition of Eq. (1) equals zero. We can get the equality of the right-hand side in Eq. (2). Because the equality can capture this tradeoff between a change in input wealth *w* and a change in weather risks *p*, the VWR is defined in Eq. (2). The higher value of VWR means that the higher impact of weather risk on the revenue and that it is worth devoting more wealth to reduce weather risk.

Observe that the VWR depends on *w*, *p*, and the revenue function conditioned on the basis of weather risk. The denominator of equality on the right-hand side in Eq. (2) corresponds to the expected marginal benefit. The marginal benefit is always positive; otherwise, the DMs are unwilling to devote more wealth into production. The larger expectation of marginal benefit leads to a smaller VWR. That is, the DM should invest more wealth into the revenue function to obtain more profit because of the probability of a higher marginal revenue or lower loss. The numerator of the equality on right-hand side in Eq. (2) corresponds to the revenue difference between under a normal and bad weather. Once the revenue difference increases, the weather risk affects the revenue or profit to a greater extent. The increase in VWR makes it worth investing more wealth into weather risk management to stabilize the revenue or profit. The formula in Eq. (1) shows the basic value of weather risk.

To analytically study ambiguity aversion, we introduce the ambiguity attitude used by Klibanoff et al. (2005) (KMM hereafter). The ambiguity attitude function ϕ is a linear one if a DM is ambiguity neutral, or a concave one if a DM is ambiguity averse, or a convex one if a DM is ambiguity preferring. Because empirical studies indicate that most people are ambiguity averse, the ambiguity attitude function ϕ is assumed to follow $(-1)^{(n+1)}\phi^{(n)} > 0$ for any positive integer *n*. For comparison, we assume that the DM's subjective beliefs are such that the expected valuation of the ambiguity random variable ε, $E[\varepsilon]$, is equal to zero. The occurrence probability with ambiguity of bad weather is denoted by $\tilde{p} = p + \varepsilon$. The DM's utility *W* is expressed using

$$W = \phi^{-1}(E[\phi((1-\tilde{p})u(w)+\tilde{p}v(w)-w)]) \tag{3}$$

where $\tilde{p} = p + \varepsilon$ and $B = (1-\tilde{p})u(w)+\tilde{p}v(w)-w$. The utility *W* in Eq. (3) is less than the expected benefit *B* in Eq. (1) because of the concave function ϕ, as shown by the subsequent inequality in Eq. (4), indicating that the subjective belief of DMs' ambiguity aversion reduces the expected benefit. The inequality $E[\phi(B)] < \phi[E(B)]$ implies

$$\phi^{-1}E[\phi(B)] = E[B] = B \tag{4}$$

In the case of ambiguity neutrality, the ambiguity attitude function ϕ become with a linear one. The utility *W* in Eq. (3) degrades to the expected benefit *B* in Eq. (1).

The natural extension of the VWR under ambiguity and ambiguity aversion conditions is obtained using Proposition 2.1. Proposition 2.2 provides an explicit formula.

Proposition 2.2 The VWR of an ambiguity-averse DM, facing the level of ambiguity captured using ε, is analytically expressed:

$$\text{VWR}_{\varepsilon} = \frac{dw}{d\tilde{p}} = \frac{(u(w)-v(w))E[\phi'(B)]}{E[\phi'(B)B']}, \tag{5}$$

where $B' = (1-\tilde{p})u'(w)+\tilde{p}v'(w)-1$.

Proof: A DM maximizes the utility of Eq. (3). The first condition equals zero, such that we obtain the following equality:

$$\frac{dW}{d\tilde{p}} = \frac{(-u(w)+v(w))E[\phi'(B)] + E[\phi'(B)B'] \frac{dw}{d\tilde{p}})}{\phi'(E[\phi(B(w))])} = 0\,,$$

which implies that the following equality holds:

$$(-u(w)+v(w))E[\phi'(B)] + E[\phi'(B)B' \frac{dw}{d\tilde{p}}] = 0\,.$$

We obtain the right-hand side of equality in Eq. (4). Because the equality combines the VWR with DM's ambiguity and ambiguity aversion, the $\mathrm{VWR}_{\varepsilon}$ is defined using Eq. (4). Under conditions of ambiguity neutrality, ϕ' is constant, such that Eq. (4) becomes Eq. (2) because $\phi'' < 0$. According to the formula in Eq. (5), an ambiguity-averse DM facing higher weather ambiguity would accept larger VWR, and then tend to pay more for reducing weather risk.

We subsequently examine the effect of ambiguity aversion on the VWR. DMs become more ambiguity averse; in that, the ambiguity aversion function φ is more concave than ϕ. We thus define φ using a concave transformation h of ϕ, where $h > 0$, $h' > 0$, and $h'' < 0$.

Proposition 2.3. An increase in ambiguity aversion always leads to an increase in $\mathrm{VWR}_{\varepsilon}$. That is, the following inequality should hold:

$$\frac{u(w)-v(w)}{B'} < \frac{(u(w)-v(w))E[\phi'(B)]}{E[\phi'(B)B']} \tag{6}$$

and
$$\frac{(u(w)-v(w))E[\phi'(\widetilde{B})]}{E[\phi'(\widetilde{B})\widetilde{B}']} < \frac{(u(w)-v(w))E[\varphi'(\widetilde{B})]}{E[\varphi'(\widetilde{B})\widetilde{B}']}\,. \tag{7}$$

Proof: As the probability $\tilde{p}$ increases, B decreases because $u > v$, and $\phi'(B)$ subsequently increases because $\phi'' < 0$. Simultaneously, B' decreases because $u' > v' > 0$. Thus, the covariance of $\phi'(B)$ and B' is negative. The covariance rule implies that the following inequality will hold:

$$E[\phi'(B)B'] < E[B']E[\phi'(B)].$$

We have

$$\frac{(u(w)-v(w))E[\phi'(B)]}{E[\phi'(B)B']} > \frac{(u(w)-v(w))E[\phi'(B)]}{E[(\phi'(B)]E[B']} = \frac{u(w)-v(w)}{E[B']}.$$

Let $\varphi = h(\phi)$ be a concave transformation h of ϕ so that the DM is more ambiguity averse; $E[\varphi'(B)] = E[h'(\phi(B))\phi'(B)]$. As the probability $\tilde{p}$ increases, $h'(\phi(B))$ increases because $h'' < 0$. The positive covariance of $h'(\phi(B))$ and $\phi'(B)$ implies that

$$E[h'(\phi(B))\phi'(B))] > E[h'(\phi(B))]E[\phi'(B))].$$

Similarly, the negative covariance of $\varphi'(B)$ and $(1-\tilde{p})u'(w) + \tilde{p}v'(w) - 1$ implies that

$$E[(\varphi'(B)((1-\tilde{p})u'(w) + \tilde{p}v'(w) - 1)] < E[(\varphi'(B)]E[(1-\tilde{p})u'(w) + \tilde{p}v'(w) - 1]\,.$$

Therefore, we have $\frac{(u(w)-v(w))E[\varphi'(B)]}{E[\varphi'(B)B']} > \frac{(u(w)-v(w))E[h'(\phi(B))]E[\phi'(B)]}{E[h'(\phi(B))]E[\phi'(B)B']} = \frac{(u(w)-v(w))E[\phi'(B)]}{E[\phi'(B)B']}$,

where the covariance of $h'(\phi(B))$ and $\phi'(B)B'$ is negative. According to Eq. (6) and (7), a DM with ambiguity aversion tends to estimate larger VWR than the one with ambiguity neutral, and is willingness to pay more for reducing weather risk.

Because the introduction of ambiguity increases the VWR of an ambiguity-averse DM compared with cases with no ambiguity, more ambiguity can also lead to greater VWR.

Proposition 2.4. A larger ambiguity ε' than ε always leads to larger $\text{VWR}_{\varepsilon'}$ of an ambiguous averser than $\text{VWR}_{\varepsilon'}$. That is, the following inequality should hold:

$$\frac{(u(w)-v(w))E[\phi'(B)]}{E[\phi'(B)B']} > \frac{(u(w)-v(w))E[\phi'(B')]}{E[\phi'(B')B']} \tag{8}$$

where $p' = p + \varepsilon'$ and $B' = (1-p')u(w) + p'v(w) - 1$.

Proof: An increase in ambiguity reduces the utility of an ambiguous averser, $\phi(B') < \phi(B)$, and therefore increases the first derivative of the concave function ϕ, $\phi'(B') > \phi'(B)$. It implies that $E[\phi'(B')] > E[\phi'(B)]$. Since the covariance between $\phi'(B)$ and B is negative, similarly, the covariance between $\phi'(B)$ and B', the one between $\phi'(B)$ and B', and the one between $\phi'(B')$ and B, are shown to be negative. The following inequality hold due to $E[B'] = E[B]$: $E[\phi'(B')((1-p')u'(w) + p'v'(w) - 1)] < E[\phi'(B')((1-p)u'(w) + pv'(w) - 1)] < E[\phi'(B)((1-p)u'(w) + pv'(w) - 1)]$.

Therefore, $\frac{E[\phi'(B)]}{E[\phi'(B)B']} > \frac{E[\phi'(B')]}{E[\phi'(B')B']}$. According to Eq. (8), the VWR become large as the ambiguity aversion of a DM gets high. Such DM tends to pay more for reducing weather risk.

When the occurrence probability of a bad weather is the main cause of increased VWR or VWR_ε, a DM may make an effort *e* to decrease this probability *p*(*e*). The convexity of *p*(*e*) with $p''(e) > 0$ is logical because *p*(*e*) reduces ($p'(e) < 0$) and the marginal decrement ($-p'(e)$) as the effort *e* decreases. The condition sufficient for self-protection is the expected benefit after self-protection exceeds what it was previously.

$$(1-p)u(w) + pv(w) - w < (1-p(e))u(w-e) + p(e)v(w-e) - w. \tag{9}$$

Eq. (9) is rewritten as $pv(w) - p(e)v(w-e) < (1-p(e))u(w-e) - (1-p)u(w)$. That is, the expected increase in normal revenue is larger than the expected decrease in bad revenue. Subsequently, the DM's objective is to select the prevention effort *e* to maximize the expected benefit:

$$B_e = (1-p(e))u(w-e) + p(e)v(w-e) - w \tag{10}$$

If there exists e^* such that the first condition of Eq. (9) equals zero, we have

$$\frac{(-p'(e^*))(u(w-e^*) - v(w-e^*))}{(1-p(e^*))u'(w-e^*) + p(e^*)v'(w-e^*)} = 1. \tag{11}$$

The existence of e^* may be logical because $(-p'(e))(u(w-e) - v(w-e))$ decreases and $(1-p(e))u'(w-e) + p(e)v'(w-e)$ increases as the effort *e* increases when the revenue difference $u(w) - v(w)$ is larger than the expected marginal benefit $(1-p)u'(w) - pv'(w) - 1$.

Proposition 2.5. The DM can reduce the VWR to the special level *a* by paying special *e* for self-protection.

$$a = \frac{u(w-e) - v(w-e)}{(1-p(e))u'(w-e) + p(e)v'(w-e) - 1} < \frac{u(w) - v(w)}{(1-p)u'(w) + pv'(w) - 1} \tag{12}$$

Proof: A DM maximizes a (state-dependent) expected benefit. The first condition of Eq. (10) equals zero. It implies the left-hand side of inequality in Eq. (12). $u(w-e) - v(w-e) < u(w) - v(w)$ holds because $u' > v' > 0$. Because *u* and *v* are concave functions, the revenue margin $u'(w-e) > u'(w)$ and $v'(w-e) > v'(w)$. This implies that $(1-p)u'(w) + pv'(w) - 1$ is larger than $(1-p(e))u'(w-e) + p(e)v'(w-e) - 1$. Thus, an effort *e* can reduce the VWR. The function *f*(*e*) defined using the following equation is continuous because that *u*(*w*) and *v*(*w*) are continuously differentiable and decrease with the effort *e*. Therefore, we can identify a special *e* such that $f(e) = a$.

$$f(e) = \frac{u(w-e) - v(w-e)}{(1-p(e))u'(w-e) + p(e)v'(w-e) - 1}$$

As ambiguity and ambiguity aversion increase, the VWR increases, as shown in Proposition 2.3. The DM can reduce the VWR to the same level *a* by taking some self-protection effort *e*.

Weather forecasting is a useful method of self-protection. If the forecast is normal, the business operations start; otherwise, they stop. The expected revenue is expressed by using $B_g = (1-p_{b|g})u(w) + p_{b|g}v(w) - w$, where $p_{b|g}$ denotes the probability of a bad weather given by a normal forecast. We assume that the weather forecast is skillful. $p_{b|g}$ is less than *p*; consequently, $1-p_{b|g}$ is larger than 1−*p*. Therefore, B_g exceeds *B*. The information value of a skillful forecast is the difference between B_g and *B*, expressed using

$$(1-p_{b|g})u(w) + p_{b|g}v(w) - w - ((1-p)u(w) + pv(w) - w) = (p - p_{b|g})(u(w) - v(w)). \quad (13)$$

Observe that the value of information increases with the accuracy of weather forecast $1-p_{b|g}$ and the difference in revenue $u(w) - v(w)$. The DM may devote effort *e* in weather forecasting to obtain a lower losing probability $p(e) = p_{b|g}$. The VWR thus decreases.

When loss from a bad weather is the main cause for the increase in the VWR or VWR$_\varepsilon$, the DM may purchase insurance to compensate for severe losses. We assume that insurance coverage and premiums are denoted by *I* and *τ*, respectively. The sufficient condition for purchasing self-insurance is that the expected benefit with insurance is larger than that without insurance:

$$B_I = (1-p)u(w-\tau) + p(v(w-\tau) + I) - w > (1-p)u(w) + pv(w) - w, \quad (14)$$

where B_I denotes the expected benefit with insurance. This implies that the expected insurance claim is larger than the expected reduction in revenue. However, this seldom is the case because the insurer obtains a negative expected benefit. Although DMs' expected benefits decrease, they may still purchase insurance because insurance can stabilize revenue volatility. This advantage contributes to business operations and lowers the probability of financial distress.

Proposition 2.6. A DM can reduce the VWR to the special level *b* by buying an insurance policy with special premium *τ* and coverage *I*:

$$b = \frac{u(w-\tau) - v(w-\tau) - I}{(1-p)u'(w-\tau) + pv'(w-\tau) - 1} < \frac{u(w) - v(w)}{(1-p)u'(w) + pv'(w) - 1}. \quad (15)$$

Proof: A DM maximizes a (state-dependent) expected benefit. The first condition of B_I in Eq. (14) equals zero, implying the left-hand side of inequality in Eq. (15).

$$\frac{dB_I}{dp} = -u(w-\tau) + v(w-\tau) - \frac{dw}{dp} + I + (1-p)u'(w-\tau)\frac{dw}{dp} + pv'(w-\tau)\frac{dw}{dp}$$

$u(w-\tau) - v(w-\tau) - I < u(w) - v(w)$ holds because $u' > v' > 0$. Because *u* and *v* are concave functions, $u'(w-\tau) > u'(w)$ and $v'(w-\tau) > v'(w)$. This implies that $(1-p)u'(w) + pv'(w) - 1$ is larger than $(1-p)u'(w-\tau) + pv'(w-\tau) - 1$. Thus, a MD can reduce the VWR by purchasing some insurance policies. The function *f* (*τ*, *I*), defined by the following equation, is continuous because *u* and *v* are continuously differentiable and decrease with premium *τ* and coverage *I*. Therefore, we can identify a specific *τ* and *I* such that *f* (*τ*, *I*) = *b*.

$$f(\tau, I) = \frac{u(w-\tau) - v(w-\tau) - I}{(1-p)u'(w-\tau) + pv'(w-\tau) - 1}.$$

DMs facing greater ambiguity or having greater ambiguity aversion have larger VWRε, as shown in Proposition 3.3. They must pay higher insurance premiums τ for high coverage I to reduce the VWR to the same level b.

3. Generality and limits of models

Consider an individual region (situation) i occupying a ratio (probability) q_i to the entire area (all situations) and facing individual weather risks with a probability p_i. The weather risk $\tilde{p}$ has a probability p_i with a ratio q_i for i =1,…,n, where $\sum_{i=1}^{n} q_i = 1$, and hence $E(\tilde{p}) = \sum_{i=1}^{n} q_i p_i = p$. DMs may require the VWR for the entire area to manage the total weather risk. On the one hand, the model generality may be applied to the various probabilities of a bad weather in different areas. The government aspires to establish compulsory disaster insurance to insure individuals against weather risk and to redistribute wealth across agents in the economy. Public utilities or services account for social equity and must maintain a uniform outcome for all people. They cannot redistribute input sources according to different weather risk exposures in different areas. On the other hand, the model generality may be applied to the different probabilities of different bad weathers. Occurrence probabilities q_i for bad weather in cases of storms, hurricanes, floods, heavy rainfall, and droughts differ. The revenue function corresponds to different weights p_i.

According to the natural extension of the model generality under ambiguity and ambiguity aversion conditions, the DM's utility is expressed using

$$W_{\tilde{p}} = \phi^{-1}\left(\sum_{i=1}^{n} q_i(\phi((1-\tilde{p}_i)u(w) + \tilde{p}_i v(w) - w))\right), \tag{16}$$

where $\tilde{p}_i = p_i + \varepsilon$ for all i =1, 2, 3,…, n. DMs with ambiguity aversion still have a greater VWR than those with ambiguity neutrality, as shown in the following Proposition. Greater ambiguity leads to identical results.

Proposition The analytical value of the multi-weather risk of an ambiguity-averse DM facing the level of ambiguity captured using ε can be expressed as follows:

$$\mathrm{VWR}_{\tilde{p}} = \frac{(u(w)-v(w))\sum_{i=1}^{n}\phi'((1-\tilde{p}_i)u(w)+\tilde{p}_i v(w)-w)}{\sum_{i=1}^{n} q_i(\phi'((1-\tilde{p}_i)u(w)+\tilde{p}_i v(w)-w)((1-\tilde{p}_i)u'(w)+\tilde{p}_i v'(w)-1)))} \tag{17}$$

and

$$\mathrm{VWR}_{\tilde{p}} > \frac{u(w)-v(w)}{(1-p)u'(w)+pv'(w)-1}, \tag{18}$$

where $\tilde{p} = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, ..., \tilde{p}_n)$ corresponds in order to the ratio series $(q_1, q_2, q_3, ..., q_n)$ and $E(\tilde{p}_i) = p_i$.
Proof: A DM maximizes the utility of Eq. (15). The first condition equals zero so that we obtain the equality:

$$\sum_{i=1}^{n} q_i(\phi'((1-p_i)u(w)+p_i v(w)-w)(((1-p_i)u'(w)+p_i v'(w)-1)\frac{dw}{dp} - \frac{dp_i}{dp}u(w) + \frac{dp_i}{dp}v(w)) = 0.$$

We obtain the right-hand side of equality in Eq. (16).

The inequality $\dfrac{(\phi'((1-p_i)u(w)+p_i v(w)-w)}{\sum_{i=1}^{n}\phi'((1-p_i)u(w)+p_i v(w)-w)} < 1$ implies the inequality

$$\frac{(\phi'((1-p_i)u(w)+p_i v(w)-w)}{\sum_{i=1}^{n}\phi'((1-p_i)u(w)+p_i v(w)-w)}q_i((1-p_i)u'(w)+p_i v'(w)-1)<q_i((1-p_i)u'(w)+p_i v'(w)-1).$$

We summarize the two sides of this inequality. Because $\sum_{i=1}^{n} q_i = 1$ and $\sum_{i=1}^{n} q_i p_i = p$, we have $\sum_{i=1}^{n} q_i((1-p_i)u'(w)+p_i v'(w)-1) = (1-p)u'(w)+pv'(w)-1$. It implies that Eq. (18) holds. Eq. (17) generalizes the application of simple VWR model with two-state revenues and one kind disaster as mentioned in Session 2.

The validity of self-protection and insurance in reducing the VWR is under the concave assumption of normal and bad revenue functions. However, in practice, these two revenue functions may not always be concave. Propositions 2.4 and 2.5 may not hold. For example, Debertin (2012) illustrates a neoclassical production function in agriculture as shown in Figure 1; this function has long been used to illustrate relationships between inputs (wealth), such as seeding, weeding, application of fertilizers, irrigation, and pesticides, and outputs (yields), such as drying, processing, and preservation. The curve is convex in the first stage but concave in the second. Therefore, the marginal revenues in the first stage increase with input wealth. Such marginal revenues may be sufficiently large to invert the inequality in Eqs. (12) and (15). That is, the revenue increases rapidly during the initial period of production, and thus, the DM should invest more wealth in production and less in self-protection and insurance to maximize the expected benefit.

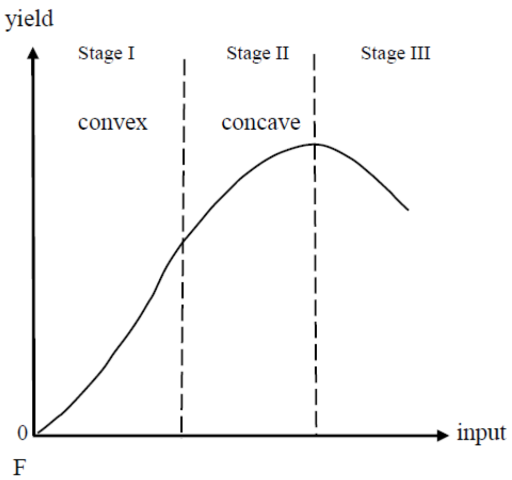

Figure 1. General agricultural production function

4. Conclusion

The frequency and intensity of extreme weather events have increased recently. Many countries have appropriated considerable funding to develop their weather distributions, particularly after sustaining severe climatic damage. Humans remain unable to control and accurately forecast the weather, and weather risks are ambiguous. Moreover, the sources of ambiguity may be complex. Every ambiguity-averse DM facing ambiguity should manage weather risk to maximize their expected benefit. They should estimate individual VWR and subsequently decide which action to take to reduce the VWR (e.g., self-protection or insurance).

To formulize the VWR and analytically study ambiguity aversion, this study assumes that weather-sensitive industries have normal and bad revenues and thus marginal revenue in normal and bad years, respectively. We derive the analytical expression of the VWR of an ambiguity-averse DM facing weather ambiguity. This VWR depends on the revenue difference and expected marginal revenue. As risk ambiguity and ambiguity aversion increase, the VWR increases; however, the VWR decreases as the use of self-protection (reducing the occurrence probability of a bad weather) or insurance (reducing the amount of loss) increases. The above-mentioned conclusion holds if the revenue function is concave. The insurance and self-protection against weather risk is inefficient if the revenue function is convex. Accurate forecasts have been widely used in self-protection. The forecast accuracy and revenue differences can be used to determine the value of weather forecasts.

The model developed herein can be generalized to evaluate the VWR under climatic multi-risk or multi-area conditions. During the convex production stage, self-production or insurance may not be DMs' optimal choices because the higher marginal yield provides DMs with more expected revenues than expected weather losses. The discussion and analytic expression of the VWR in this paper contribute to benefit–cost analysis in weather risk management.

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